Should we teach Total Factor Productivity?

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1 Introduction

In the preamble of the Economic and Social Sciences (ESS) programme for students between 15 and 18 years old, lately revised in 2019, it is said that "teachers must insist on the requirement for axiological neutrality. Social sciences rely on established facts, rigorous argumentation, validated theories and not on values. The object of ESS teaching is the result of scientific work, transposed to educational learning. It should help students to make a clear distinction between scientific approach and scientific knowledge on the one hand, and what is a matter of faith or dogma on the other hand" (ESS Programme, 2019).

The first chapter of the programme of the Terminale class - for students between 16 and 18 years old - is the study of the sources and challenges of economic growth. We find the notion of Total Factor Productivity (TFP) on the following specification:

> "To understand the process of economic growth and the sources of growth : accumulation of factors and increase of total factor productivity ; understand the link between technical progress and increase of the total factor productivity" (ESS Programme, 2019).

As we will see, the concept of Total Factor Productivity (TFP), while currently used by many economists worldwide, is actually inaccurate, has been invalidated several times in scientific papers and the fact that it is still in use seems to be based on reasons that have more to do with faith or dogma than established facts, rigorous argumentation and validated theory.

This raises important questions about what should we do as teachers : in one hand, it is mandatory to teach the programme, with all its specifications ; on the other hand, we should teach only validated theories, as underlined by the preamble of the programme. The goals of this article is to explain why TFP does not measure what it ought to measure, to explain why the original mistake of the economists that have introduced TFP was not corrected and finally to look at possibilities to change this aspect of the programme in order to avoid teaching non scientific notions to students.

2 Why TFP is to be abandoned

Total factor productivity is famous since the publication in 1957 of an article by Robert Solow. It is identified by Solow as the part of economic (measured by growth of Gross Domestic Product - GDP) which is not explained by the growth of the factors of production that are capital and labour. This unexplained "residual" is then called the growth of total factor productivity, which Solow interprets as the measure of technical progress. We will show here how is calculated TFP and why it does not measure what it ought to measure. Our reasoning here is mainly based on a book from Jesus Felipe and John McCombie (2013) and an article from Bernard Guerrien and Ozgur Gun (2014).

2.1 How is measured TFP?

How is measured total factor productivity? There is two ways to consider this. First, what economists think they are doing, second what they are actually doing.

2.1.1 What economists think they are doing

Economists working on neoclassical growth theory (such as James Cobb and Paul Douglas, "inventors" of the Cobb-Douglas production function, and Robert Solow) start their analyses by assuming 3 hypotheses:

(1) There is, at the national level, an aggregate production function such that:

$$Y = f(L, K) \tag{1}$$

With Y the value of GDP (measured in price terms), L the quantity of labour (for instance measured in hours of labour) and K the value of the capital (measured in price terms).

(2) The labour factor is remunerated at its marginal productivity, i.e. the derivative of the aggregate production function with respect to labour change is equal to the wage rate:

$$\frac{\Delta Y}{\Delta L} = \frac{\Delta f(L, K)}{\Delta L} = w \tag{2}$$

With w the wage rate (average wage for one hour of labour, is labour is measured in hours of labour).

(3) The capital factor is remunerated at its marginal productivity, i.e. the derivative of the aggregate production function with respect to capital change is equal to the profit rate:

$$\frac{\Delta Y}{\Delta K} = \frac{\Delta f(L, K)}{\Delta K} = \pi \tag{3}$$

With π the profit rate (without unit, Solow sometimes calls it the cost of location of capital and we can approximate it with the average rate of interest).

With these three hypotheses, which are common in neoclassic theory, economists working on the neoclassical growth theory show that the growth of GDP can entirely be explained by the growth of labour, the growth of aggregate capital, and a "residual":

$$\Delta Y = w\Delta L + \pi\Delta K + reste \tag{4}$$

This is this "residual" that Solow calls growth of total factor productivity and that ought to measure the influence of technical progress on economic growth.

James Cobb and Paul Douglas decided, in 1928, to adjust an aggregate production function with the form $f(L, K) = AL^{\alpha}K^{\beta}$ to the data they had collected on GDP, labour and capital in the United States between 1899 and 1922. This function, now called Cobb-Douglas, is in accordance with the three hypotheses above. To their great pleasure, they found that when estimations of A, α and β were giving the best GDP results, it was with values concerning marginal productivities of wage and capital close to the wage and profit rates empirically observed. This was the start of the now widely used aggregate production functions with aggregate capital measured in price terms, and where marginal productivities of labour and capital are assimilated to wage and profit rates.

2.1.2 What the economists are really doing

Production functions with several inputs have been criticized, both at the level of an individual firm (for instance, see Moseley, 2015) and at the level of a nation. More precisely, the fact that we could calculate marginal productivities independently of prices has been rejected during the

controversy in the 1960s between the two Cambridge (a controversy with among participants were Paul Samuelson, Robert Solow, Joan Robinson, Piero Sraffa and Luigi Pasinetti). Thus the neoclassical theory of distribution, linking revenues of labour and capital with their marginal productivities, has been put in severe doubt (Geoff Harcourt, 1976). However, Cobb, Douglas and Solow indeed found empirical results in accordance with hypotheses (1), (2) and (3): it seemed possible to reproduce the fluctuations of GDP by constructing an aggregate production function where marginal productivities of labour and capital are equal to the wage and profit rates that are really measured empirically.

However, as we will see, there is no need of hypotheses (1), (2) and (3) to find equation (4).

Knowing that the value of GDP is always equal to the sum of the revenues (this is an equality which is always verified, thus independent from economic theories: it is an accounting identity), we can write:

$$Y = wL + \pi K \tag{5}$$

Now, this accounting identity is in accordance with the hypotheses (1), (2) and (3)! Starting from this accounting identity, growth of GDP can be broken down into the growth of labour, the growth of capital and the variations of the wage and profit rates:

$$\Delta Y = w\Delta L + \pi\Delta K + L\Delta w + K\Delta\pi \tag{6}$$

It is then sufficient to say that the last two terms are an unexplained "residual" to find back equation (4), with:

$$residual = L\Delta w + K\Delta\pi \tag{7}$$

The residual, called by Solow "the growth of total factor productivity", appears then to be a measure of the variation of wage and profit rates, and not a measure of the influence of technical progress on growth.

The theoreticians of neoclassical growth thought to have contributed to explain the causes of economic growth (growth is caused by the growth of the elements of an aggregate production function where factors are remunerated according to their marginal productivities), whereas they were only manipulating an accounting identity, which does not explain anything (all theories are compatible with equation (5)). And it is not surprising that their results where corroborated by empirical observations, as their point of departure was an always true accounting identity. They were only transforming it into an other equation, more complicated.

This can be shown with the Cobb-Douglas aggregate production function. It is possible to rewrite the distribution of national revenue in the following way:

$$wL + \pi K = \left[\alpha^{-\alpha}\alpha^{\alpha-1}w^{\alpha}\pi^{1-\alpha}\right] L^{\alpha}K^{1-\alpha} \tag{8}$$

With the only condition that $\alpha = \frac{wL}{wL+\pi K} = constant$, i.e. with the condition that the share of national revenue given to labour is constant over time. It is then sufficient to simplify equation (8) by writing:

$$A = \left[\alpha^{-\alpha}\alpha^{\alpha-1}w^{\alpha}\pi^{1-\alpha}\right] \tag{9}$$

To obtain the Cobb-Douglas production function :

$$wL + \pi K = AL^{\alpha}K^{1-\alpha} \tag{10}$$

This function will often pass empirical tests, because it is a reformulation of an always verified accounting identity.

It is true, however, that this reformulation is only valid if the partition of national revenue between labour and capital is constant over time. But this was approximately true in the industrial countries during the XXth century. The transformation of the accounting identity into a Cobb-Douglas function was then often possible, which has contributed to the success of this "explanation" of economic growth.

3 Why TFP is part of the programme and how to cope with it

3.1 How is it possible that TFP is part of the programme?

How is possible that economists did not realise that they were manipulating an accounting identity?

Firstly, this problem was discussed. Numerous scientific articles published in prestigious journals described the problem and clearly showed were was the mistake, just after Solow's publication: Phelps Brown (1957), Warren Hogan (1958), Herbert Simon and Ferdinand Levy (1963), Anwar Shaikh (1974). Herbert Simon in 1978, during the conference he gave while receiving the prize from the Bank of Sweden in the memory of Alfred Nobel, warned again that using a Cobb-Douglas function was misleading, as it was a reformulation of an accounting identity (Simon, 1979). In 2000, James Hartley used a Real Business Cycle model with a Cobb-Douglas function. In these kind of models, exogenous shocks are applied to the model to register its reaction. In Hartley's model, exogenous technical changes were modeled and Hartley showed that TFP was not measuring the technical change applied to the model. Recently, a succession of articles by Jesus Felipe and John McCombie, which were edited in one book in 2013, dealt with this problem in depth.

However, TFP was and still is widely used, as well as the neoclassical theory of growth. If textbooks in the 1970s presented both the neoclassical theory of growth and other alternative theories (for instance the post-keynesian model from Nicholas Kaldor), these textbooks will progressively get rid of these alternative theories in the 1980s. Two explanations can be found for this curious lack of efficiency of the scientific process in getting rid of mistakes.

3.1.1 The empirical argument : a matter of faith

Firstly, the results founded were so close to empirical observations that economists decided to carry on regardless the criticism on the method. Results were so good that doubts must be dismissed, as an act an faith. This is clearly stated by Gregory Mankiw, author of one of the most famous neoclassical textbooks:

> ", I have always found the high R2 reassuring when I teach the Solow growth model. Surely, a low R2 in this regression would have shaken my faith that this model has much to teach us about international differences in income" (Mankiw, 1997, p. 104)[1]

3.1.2 The theoretical argument : a matter of dogma

Secondly, results seemed to confirm the neoclassical theory of distribution, and this was really important, as it is clearly stated by Paul Douglas during a conference in 1976:

[1] R^2 is an index whose value, between 0 and 1, indicates whether theoretical results are more or less close to empirical data (respectively R^2 close to 1 or R^2 close to 0).

> "A considerable body of independent work tends to corroborate the original Cobb-Douglas formula, but more important, the approximate coincidence of the estimated coefficients with the actual shares also strengthens the competitive theory of distribution and disproves the Marxian." (Douglas, 1976).

As the profit rate seems to be equal to the marginal productivity of capital, this would mean that capitalists are earning the wealth they are producing. This is the core of the neoclassical theory of distribution. And if validated criticisms has been raised since, they have been forgotten because they were not in accordance with the theory. Now this is a characteristic of a dogmatic position that facts that are in accordance with it are proudly claimed, while facts that are against it are simply ignored.

Total factor productivity is then a concept which does not measure what it ought to, and which is in the programme of ESS in France because of reasons rather concerning faith and dogma than established facts, rigorous argumentation and validated theory.

3.2 What can be done for the teacher?

If it is important that non-scientific concepts get rid of the programme, such as TFP, this does not mean that students should not learn the causes of economic growth. Furthermore, if the models and their conclusions are wrong, this is not always the case of the intuitions behind them.

3.2.1 Changing the programme

Here are the specification of the current programme for the chapter on economic growth for the Terminale class:

- To understand the process of economic growth and the causes of growth : accumulation of factors and increase of total factor productivity ; understand the link between technical progress and increase of the total factor productivity.
- To understand that technical progress is endogenous and that, in particular, it comes from innovation.
- To understand how institutions (especially property rights) impact growth through incentives to invest and innovate ; to know that innovation brings with it a process of destructive creation.
- To understand how technical progress can generate revenue inequalities.
- To understand that a sustainable economic growth will be confronted to ecological limits (especially exhaustion of resources, pollution and global heating) and that innovation can help to make those limits less constraining.

Only the first item needs to be changed, for instance like this:

- To understand the process of economic growth and the causes of growth: accumulation of factors and increase of labour productivity; to understand the link between technical progress and increase of labour productivity.

Indeed, the important mechanism to understand is that growth can result from an increase of the quantity of factors used in production, or can result from a better productivity, an increase in productivity being the measure of technical progress. This can be done by relying only the labour factor, by saying that an increase in the productivity of labour can have several

causes, such as better qualification of employees, purchase of more efficient machinery, or a better organisation of the work. In fact, working with a function of production with only one input, labour, and studying the evolution of its quantity and its productivity is the method chosen by John Maynard Keynes in his General Theory (1936). This solution is also the one recommended by Angelo Reati (2001) if we want to avoid using TFP and the french National Institute for Statistics and Economic Studies (INSEE) uses a production function with labour as the only factor in some of its studies.[2]

3.2.2 An example of what is currently done with the students

Activity : Measuring the causes of growth

Annual variation of GDP (in %) and contributions to growth (in points of %)

New-Zealand	2015	2016
Economic growth (variation of GDP in volume) in %	4.2	3.8
Contribution to growth from the labour factor in points of %	1	3.2
Contribution to growth from the capital factor in points of %	1	0.8
Contribution to growth from total factor productivity in points of %	2.2	...

Source : OECD database, 2021

Q1. How do we get the underlined data?

Answer: In New Zealand, in 2015, according to OECD, GDP increase was about 4.2%, compared to the previous year. This number is the sum of the contributions from the different factors: $1 + 1 + 2.2 = 4.2$.

Q2. Calculate the contribution to growth from TFP in 2016.

Answer : - 0.2 points of %.

The contribution to growth from TFP in New-Zealand in 2016 is negative: $3.8 - 3.2 - 0.8 = -0.2$ %. We can interpret this number by saying that the technology or the organisation of the work were less efficient in 2016 in New Zealand, compared to the previous year.

Q3. Show that economic growth does not have the same causes in 2015 and 2016.

Answer : We notice that in 2016 the labour factor is an important cause of economic growth because 3.2 points can be explained by this factor whereas the economic growth is only about 3.8 %. On the contrary, in 2015, the TFP is the most important cause of economic growth: it explains 50% of the economic growth of New-Zealand.

This is a very classical exercise that we can find in most textbooks. TFP appears like something natural and as an important cause of economic growth. However, as seen in equation (7), the growth of TFP is actually equal to $L\Delta w + K\Delta\pi$ and, thus, does not explain anything.

[2] Cf. the working paper from Matthieu Lequien and Alexis Montaut available at the following URL: http://www.insee.fr/fr/publications-et-services/docs_doc_travail/G2014-09.pdf

3.2.3 An example of what could be done instead with the students

We could do the same exercise by taking into account only the labour factor, measured in hours of labour:

Annual variation of GDP (in %) and variations of labour and labour productivity (in %)

New-Zealand	2015	2016
Economic growth (variation of GDP in volume)	4.4%	3.8%
Growth of labour factor (in hours)	1.9%	4.8%
Growth of labour productivity (GDP/hours of labour)	2.5 %	... %

Source : OECD database, 2021

Q1. How do we get the underlined data?

Answer: In New Zealand, in 2015, according to OECD, GDP increase was about 4.4%, compared to the previous year. This number is the sum of the contributions from the growth of the quantity of labour used and the growth of labour productivity: $1.9 + 2.5 = 4.4$.

Q2. Calculate the growth of labour productivity in 2016.

Answer : -1 %.
The growth of labour productivity in 2016 is negative: $3,8 - 4,8 = -1$ %. We can interpret this number by saying that the technology or the organisation of the work were less efficient in 2016 in New Zealand, compared to the previous year.

Q3. Show that economic growth does not have the same causes in 2015 and 2016.

Answer : We notice that in 2016 the labour factor is an important cause of economic growth because its growth is more important than the growth of GDP. On the contrary, in 2015, the growth of labour productivity was the most important cause of economic growth: it explains 57% of the economic growth in New-Zealand.

As we can see, even with one factor, it is possible to distinguish several causes of economic growth, from the increase of the factors used to the increase of productivity. Data on the growth of hours of labour and labour productivity can be found for a good number of industrialised countries on the OECD database.

4 Conclusions

Total factor productivity is a concept that does not measure the growth of technical progress. It is still used by the vast majority of economists because it is calculated with the help of an aggregate production function that gives good empirical results and that confirms the neoclassical theory of distribution. However, this good empirical results are caused by the fact the aggregate production function is a reformulation of an accounting identity. The fact that this mistake was discovered long time ago but without any impact on the use of aggregate production functions and the notion of total factor productivity is clear sign that dogma and faith are sometimes more strong than logic and science.

As teacher, we are bind to teach the programme; but the preamble of the programme says that we should not teach faith or dogma. Here I showed that we can explain the different causes of economic growth without referring to TFP, but using instead labour and labour productivity. Hence, it is not necessary to teach total factor productivity.

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